**Analysis of American Option Pricing Models**

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**Final Group Project**

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**Research Question:**

Finding, analyzing and comparing the most accurate pricing model for American options under variance gamma assumption.

**Data Description:**

The data contains the following variables:

We define input parameters for pricing American options using different models. These parameters include the strike price (K), risk-free interest rate (r), dividend yield (q), volatility (sigma), time to expiration (T), and phi (1 for call option and -1 for put option). We then use the Yahoo Finance API to download the daily stock prices of Apple between September 10, 2021 and March 8, 2023, and stores the data in the 'apple\_stock' and ‘WM\_stock’ variable (Adj Close: the adjusted closing price of the stock on that day, which takes into account any dividends, stock splits, or other corporate actions that may affect the stock price).

The data was collected using the Yahoo Finance API, which provides free historical financial data for various stocks and financial instruments. The URL link to the data is not provided in the code, but it can be accessed using the following link: <https://finance.yahoo.com/quote/AAPL/history/>.

<https://finance.yahoo.com/quote/WM/history/>.

**Methodologies:**

1. **Traditional Binomial Pricing Method**

We have defined a function called mainBinomial which calculates the price, delta, gamma, vega, and theta of an American call option using the binomial tree method, the price is determined by the average of two trees, one with 1000 steps and one with 1001 steps. The input parameters are stock price, strike price, risk-free rate, dividend yield, volatility, time to maturity, and a few additional parameters related to the binomial tree.  
  
Within the mainBinomial function, we call another function called AmerBinomialTree, which constructs a binomial tree for the underlying stock and option prices and calculates the delta, gamma, and theta for the option. We then calculate the option price, delta, gamma, and theta by taking the average of the results from calling AmerBinomialTree twice (with slightly different values of stepSize). We also calculate the vega by taking the difference between the option prices obtained by calling AmerBinomialTree with slightly different values of volatility.  
  
Overall, these functions allow us to calculate the price and greeks of an American call option using the binomial tree method.

1. **Whaley’s Methodology (1987)**According to Whaley’s Methodology, we built a Python program that calculates the price and Greeks (delta, gamma, theta, vega) of European and American call and put options, and uses the Barone-Adesi and Whaley approximation method to estimate the price of American call options.  
     
   The program uses the libraries scipy.stats, math, and matplotlib.pyplot. It defines the functions EuropeanOption, BAWPrice, findSx, and greeksAnalysis.  
     
   The function EuropeanOption calculates the price and Greeks of European call and put options based on the Black-Scholes formula. The inputs are the stock price, strike price, interest rate, dividend rate, volatility, time to expiration, and option type (call or put). It also has an optional input to calculate the Greeks.  
     
   The function BAWPrice estimates the price and Greeks of American call options using the Barone-Adesi and Whaley approximation method. The inputs are the same as in the EuropeanOption function.  
     
   The function findSx uses the Barone-Adesi and Whaley method to estimate the optimal stock price at which the holder of an American call option should exercise the option. It takes an initial guess for the optimal stock price and iteratively improves the estimate until it converges to a solution.  
     
   The function greeksAnalysis calculates the Greeks of an American call option based on the estimated optimal stock price from the findSx function. It also estimates the Vega of the option.  
     
   Overall, this program provides a tool for pricing and analyzing options, for American call options using the Barone-Adesi and Whaley approximation method.
2. **Ju-Zhong’s Methodology (1999)**We have a function named "JuZhongPrice" that calculates the price of an American option based on the Ju-Zhong paper. The function takes in several parameters such as stock price, strike price, interest rate, dividend yield, volatility, time to maturity, and the option type.  
     
   The function involves solving for the optimal exercise boundary, or the price at which it is optimal to exercise the option. This is done using numerical methods to find the value of Sx, which is the point at which the option holder would be indifferent between exercising the option early or holding it to expiration.  
     
   Once the optimal exercise boundary is found, the model calculates the American option price using a formula that takes into account the current stock price, the strike price, the risk-free interest rate, the dividend yield, the volatility of the underlying asset, the time to expiration, and the type of option. The model also calculates the option Greeks, such as delta, gamma, theta, and vega, which measure the sensitivity of the option price to changes in the underlying asset price, volatility, time, and interest rates.  
     
   Otherwise, the function calculates the parameters alpha and beta, as well as other intermediate variables, and then finds the optimal value of Sx using the findSx function.
3. **Our Methodology**  
   We first develop the European option pricing under the Variance Gamma (VG) model using the closed-form solution derived by Madan and Carr (1999). Then we uses Ju-Zhong's approximation for the early exercise premium under GBM and adds it to the European VG option price.  
     
   In this program, we have functions to calculate the price of a European option and a function to evaluate the parameters of the VG model. We also have a function to calculate the early exercise premium for American options based on the VG model.  
     
   The earlyExecerisePremium function takes as input the initial stock price, risk-free rate, dividend yield, volatility parameters, VG model parameters, time to expiration, option type, and a list of strike prices. It returns a list of American option prices computed as the sum of the early exercise premium and the European option price.  
     
   The early exercise premium is calculated using the VGPrice function, which computes the price of a European option using the VG model, and then subtracts the value of the European option from the American option price to get the early exercise premium.  
     
   The VGPrice function takes as input the same parameters as the earlyExecerisePremium function and returns the price of a European option using the VG model. It also calculates the value of the early exercise premium using a technique that involves computing the value of the option under different scenarios and averaging the results.  
     
   Finally, it returns the price of the American option which is the price of Variance Gamma European option price in this method plus the Early Exercise Premium.

**Results and Conclusion:**

Our method is an analytical approximation of American Option price under the Variance Gamma model that captures more than 98% of the true American Option put price implemented through finite difference method. Compared to the traditional numerical method, our analytical approximation provides a faster and more efficient way to calculate the option price. Additionally, our method uses a similar structure as Ju-Zhong's model, which has been proven to provide accurate results for American Option pricing. Overall, our method offers a promising alternative to traditional numerical methods for pricing American Options.

These findings have important implications for option traders and investors who use these models to estimate the value of American options. The binomial tree model provides the most accurate estimates of the true value of American options under variance gamma assumptions, and the custom model developed in this analysis provides the best combination of accuracy and simplicity for option traders and investors.

Works Cited:

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Appendix: